The Multiple and Plural Scattering of Fission Fragments

D. KERR *, G. SIEGERT, K. KÜRZINGER, E. KONECNY, and H. EWALD

II. Physikalisches Institut der Universität Gießen

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The multiple and plural scattering distributions of fission fragments in Au, Ag and Formvar foils have been measured. The value of the BORN parameter in this experiment is more than an order of magnitude larger than values that have previously been involved in such measurements. Since the fission fragments are only partially ionized, some approximation must be made to account for the additional electronic screening. Two different models, which may have some application to other experiments involving partially ionized particles are presented. The experimental distributions are compared with those predicted by the theories of MOLIÈRE and of NIGAM, SUNDARESAN and WU and only with the theory of MOLIÈRE is it possible to obtain reasonable and consistent agreement with experiment.

There are two basically different theories describing small angle multiple scattering, one due to MOLIÈRE [1, 2], and the other due to NIGAM, SUNDARESAN and WU [3]. These theories predict essentially the same results for small values of the BORN parameter $\alpha = Z_1 Z_2/137\,\beta$, but for large values of this parameter, i. e., when the first BORN approximation is no longer valid, the predictions of the two theories are very different. In the definition of α, Z_1 and Z_2 are the nuclear charge numbers of the scattered and target atoms respectively, and β is the ratio of the velocity of the scattered particles to the velocity of light.

The calculation of the multiple scattering distribution of a beam of charged particles requires some knowledge of the single scattering law. This is given by some modification of the RUTHERFORD law including, most importantly, the effect of the screening of the nuclear COULOMB field of the target atom by electrons.

To account for this screening, MOLIÈRE approximates the THOMAS–FERMI potential by

$$V(r) = \frac{Z_1 Z_2 e^2}{r} \sum_{i=1}^{3} a_i \exp(-b_i r/r_0) \qquad (1)$$

where e is the electronic charge, a_i and b_i are constants, and r_0 is the THOMAS–FERMI radius given by

$$r_0 = 0.885\, a_0 Z_2^{-1/3} \qquad (2)$$

in which a_0 is the BOHR radius. Using this potential, MOLIÈRE calculates $q(\chi)$, the ratio of the single scattering cross section with screening to the RUTHERFORD cross section for an unscreened COULOMB field as a function of the scattering angle, χ, and for various values of α. For small scattering angles, where the impact parameter is large and the screening therefore most effective, $q(\chi)$ approaches zero, while for the largest scattering angles, which result from the passage of the impinging particle through an unscreened region of the atom, $q(\chi)$ becomes unity. This general behaviour of $q(\chi)$ is important in the development of the theory, and the actual form of $q(\chi)$ determines the screening angle, a parameter which enters into the determination of the shape of the multiple scattering distribution. This screening angle, χ_a, is defined by MOLIÈRE through the relation

$$\ln \chi_a = -\frac{1}{2} - \lim_{\chi_m \to \infty} \left[\int_0^{\chi_m} \frac{q(\chi)}{\chi} \mathrm{d}\chi - \ln \chi_m \right]. \qquad (3)$$

MOLIÈRE then calculates χ_a for $\alpha = 0$ and for large α and by a method of interpolation obtains the result

$$\chi_a{}^2 = \chi_0{}^2 (1.13 + 3.76\,\alpha^2) \qquad (4)$$

where

$$\chi_0 = \hbar / p\, r_0 \qquad (5)$$

in which p is the momentum of the scattered particle.

NIGAM, SUNDARESAN and WU (hereafter referred to as NSW) object to MOLIÈRE's development on the basis that an inconsistent approximation in all orders of α, except the lowest, occurs in the derivation. NSW account for the electronic screening in a

* National Research Council of Canada Postdoctoral Fellow. Present address: Department of Physics, Harvard University, Cambridge, Massachusetts.

[1] G. MOLIÈRE, Z. Naturforschg. **2 a**, 133 [1947].
[2] G. MOLIÈRE, Z. Naturforschg. **3 a**, 78 [1948].
[3] B. P. NIGAM, M. K. SUNDARESAN, and TA-YOU WU, Phys. Rev. **115**, 491 [1959].

simpler manner through the use of the exponentially screened potential

$$V(r) = (Z_1 Z_2 e^2/r) \exp(-\mu r/r_0) \quad (6)$$

where μ is a factor of order unity which is to be adjusted so that this potential best approximates the true potential. The screening factor, $q(\chi)$, is calculated using the second BORN approximation result of DALITZ [4] for the single scattering cross section of a relativistic DIRAC particle. The resulting expression for the screening angle in this case is [5]

$$\chi_a^2 = \chi_\mu^2\{1 - 4\,\alpha\,\chi_\mu[(1-\beta^2)\ln\chi_\mu + 0.2310 + 1.448\,\beta^2]\} \quad (7)$$

where $\chi_\mu = \mu\,\chi_0$.

Since μ is of order unity, and, since $\alpha\,\chi_0 \ll 1$ (SCOTT [6] shows that this is a general requirement for the validity of the small angle approximation used in the theories), it is seen from Eq. (7) that χ_0 and the NSW χ_a will always have approximately the same value. On the other hand, in MOLIÈRE's derivation, χ_a is seen to depend directly on α when α is large. It is the difference in these values of χ_a that causes the major difference in the predictions of the two theories. This discrepancy has been discussed by SCOTT [6] and by SIMON [7]. In particular, SIMON shows that MOLIÈRE's result for χ_a is correct in the classical limit, i. e., for large α.

Prior to the works of BEDNYAKOV et al. [8–10] and SIMON [7], most multiple scattering experiments had been done with electrons or positrons, therefore involving values of α ($\lesssim 0.6$) for which the NSW and MOLIÈRE theories predict approximately the same result. BEDNYAKOV et al. [8, 9] measured the scattering of protons in Cu, Al and polystyrene foils where the maximum value of α was about 20. Direct application of MOLIÈRE's theory yielded distributions in agreement with the experimental results for Al and Cu, and agreement was obtained in the case of polystyrene by replacing the THOMAS–FERMI potential used in the MOLIÈRE theory with a more accurate one obtained by the HARTREE–FOCK method. The distributions of 4 MeV N^{14} ions and 5 MeV O^{16} ions scattered in Al ($\alpha \cong 30$) were also measured [10] and were found to be in approximate agreement with those calculated on the basis of MOLIÈRE's theory using an approximation consisting of the replacement of the square of the nuclear charge of the scattered particles by their measured mean squared ionic charge.

SIMON measured the multiple scattering distributions of 164 MeV O^{16} ions and 400 MeV Ar^{40} ions in Au, Ni, Al, and Zapon foils with values of α ranging from 2.9 to 30.8. He found that MOLIÈRE's theory accurately reproduced the experimental results, whereas the theory of NSW predicted distributions that were as much as 60% too wide.

In the work reported here, fission fragments selected according to mass, energy and ionic charge were scattered in Au, Ag and Formvar foils with an experimental arrangement designed to measure the projected angle scattering distributions. The maximum value of α involved is 848. We have, therefore, the opportunity of comparing the scattering theories of MOLIÈRE and NSW with experiment for values of α that are from 1 to 2 orders of magnitude larger than previous experimental values.

Theoretical Scattering Distributions

In addition to the difference in χ_a according to the theories of MOLIÈRE and NSW, the latter theory also contains additional terms accounting for spin dependence and relativity effects. With the parameters involved in this experiment, these terms are negligible, and, except as noted below, the distributions of the projected angle, Θ, are given in both cases (using the appropriate definition for χ_a) by the same expression:

$$f(\Theta) = f^{(0)}(\vartheta) + \frac{f^{(1)}(\vartheta)}{B} + \frac{f^{(2)}(\vartheta)}{B^2} \quad (8)$$

where B is defined by the transcendental equation

$$B - \ln B = \ln \Omega_0 - 0.1544\,. \quad (9)$$

[4] R. H. DALITZ, Proc. Roy. Soc. London **A 206**, 509 [1951].

[5] This equation appears in ref. [3] with a plus sign before the term $4\,\alpha\,\chi_\mu$ since the scattering of electrons is under consideration. For the scattering of positive ions, the minus sign must be used.

[6] W. T. SCOTT, Rev. Mod. Phys. **35**, 231 [1963].

[7] W. G. SIMON, Phys. Rev. **136**, B 410 [1964].

[8] A. A. BEDNYAKOV, A. N. BOYARKINA, I. A. SAVENKO, and A. F. TULINOV, Zh. Eksperim. Teor. Fiz. **42**, 740 [1962]; Soviet Phys.—JETP **15**, 515 [1962].

[9] A. A. BEDNYAKOV, V. N. DVORETSKII, I. A. SAVENKO, and A. F. TULINOV, Zh. Eksperim. Teor. Fiz. **46**, 1901 [1964]; Soviet Phys.—JETP **19**, 1280 [1964].

[10] A. A. BEDNYAKOV, V. S. NIKOLAEV, A. V. RUDCHENKO, and A. F. TULINOV, Zh. Eksperim. Teor. Fiz. **50**, 589 [1966]; Soviet Phys.—JETP **23**, 391 [1966].

Ω_0 is a measure of the total number of scattering events and is defined as

$$\Omega_0 = \chi_c^2 / \chi_a^2 \tag{10}$$

where $$\chi_c^2 = \frac{4\pi N t e^4 Z_1^2 Z_2^2}{p^2 v^2}. \tag{11}$$

N is the number of scattering atoms per unit volume, t the thickness of the scattering foil, e the electronic charge, p the momentum and v the velocity of the impinging particle. In Eq. (8), $\vartheta = \Theta / \chi_c \sqrt{B}$. The functions $f^{(0)}$, $f^{(1)}$ and $f^{(2)}$ are tabulated by MOLIÈRE [2] but we have used the more accurate table of SCOTT's [6] "D" functions which bear a simple relationship to MOLIÈRE's "f" functions.

The values of χ_c and χ_a must be corrected to allow for energy loss in the scattering foil and to account for the mixture of elements in the case of Formvar. These corrections are made according to MOLIÈRE [2] and SCOTT [6].

As a result of the approximations in his derivation, MOLIÈRE states that his theory is valid only for $B > 4.5$ or $\Omega_0 > 20$. Therefore, when $\Omega_0 < 20$, we use the results of KEIL et al. [11] who have calculated the plural scattering distributions for the average number of scattering events in the range from 0.2 to 20. It must be emphasized however, that the proper application of their results requires a knowledge of the actual number of scattering events that a particle experiences on its passage through a foil. The relationship of their results to MOLIÈRE's theory is through an approximation to $q(\chi)$ suggested by MOLIÈRE, namely

$$q(\chi) \cong \chi^4 / (\chi^2 + \chi_a^2)^2 \tag{12}$$

which yields the ratio χ_c^2/χ_a^2 as the theoretical number of scattering events. However, for large α, Eq. (12) is a poor approximation to the exact form of MOLIÈRE's $q(\chi)$ in that it approaches zero too rapidly. Therefore, χ_c^2/χ_a^2 is smaller than the number of scattering events that would be predicted on the basis of the actual form of $q(\chi)$. By itself, the smaller number of scattering events causes a decrease in the width of the calculated distribution, but the approximation to $q(\chi)$ also affects the definition of the reduced angle used by KEIL et al. in a manner tending to produce a wider distribution. Since the two effects are relatively small and are in opposition, the net result should not be too much in error.

KEIL et al. tabulate the results only for spatial angle scattering distributions from which we have obtained the corresponding distributions in projected angle by a process of numerical integration.

As a result of the smaller χ_a in the NSW theory, the corresponding Ω_0 is always large and the scattering distributions may be obtained from Eq. (8) in each case.

Experimental

The experimental arrangement is shown schematically in Fig. 1. The source of fission fragments was a thin uranium foil mounted near the core of the reactor. The fission fragments emerge from the foil with very nearly their initial kinetic energies and with ionic charge numbers varying from about 20 to 25. A small fraction of these ions enter a MATTAUCH–HERZOG type mass spectrometer [12] consisting of the electric field of a toroidal condenser and a homogeneous magnetic field. This field combination selects particles with a particular ratio of mass to ionic charge and at the same time an actual mass separation is achieved as a result of the kinematics involved in the fission process [13]. Slits were installed in the plane of the magnet at its entrance and exit to provide a vertically well collimated beam. The scattering foil was mounted directly behind the exit slit and a nuclear emulsion plate was placed at a further distance of 280 mm, and mounted in a manner such that the scattered particles were incident at a 45° angle in order to facilitate track recognition later under the microscope. Because of low intensity, it was necessary to irradiate each plate for approximately 100 hours. After development, fission fragment tracks were counted in narrow strips on the plate parallel to the exit slit of

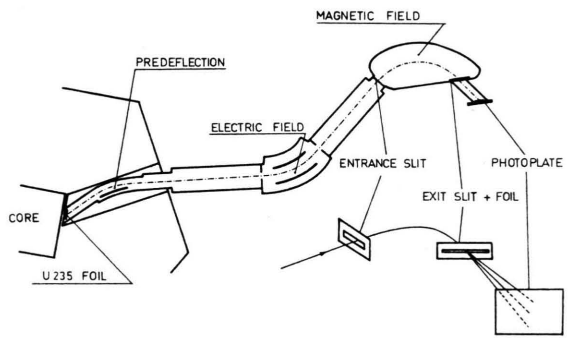

Fig. 1. Schematic diagram of the experimental arrangement.

[11] E. KEIL, E. ZEITLER, and W. ZINN, Z. Naturforschg. **15a**, 1031 [1960].

[12] H. EWALD, E. KONECNY, H. OPOWER, and H. RÖSLER, Z. Naturforschg. **19a**, 194 [1964].

[13] E. KONECNY, H. OPOWER, and H. EWALD, Z. Naturforschg. **19a**, 200 [1964].

the magnet. Since the exit slit was substantially longer than the length of the scanned strip, the experimental projected angle scattering distributions were obtained directly.

The horizontal slit lengths were such as to allow the simultaneous passage of fission fragments in a 3% mass interval which we chose to be centred at mass 135. The weighted average of the primary nuclear charge numbers in this interval is 52.5 [14] and, initially, all of the selected particles had an energy of 81 MeV and ionic charge number of 24. After passage through the foil, their energies ranged from 65 to 78 MeV depending on the thickness and atomic number of the scattering material, and the average ionic charge number was approximately 24 [15]. Scattering distributions were measured for the following foils: gold (0.315 mg/cm^2 and 0.63 mg/cm^2), silver (1.16 mg/cm^2) and Formvar (0.22 mg/cm^2).

Results and Discussion

1. The effective charge of the fission fragments

Since the fission fragments are not fully ionized but have retained approximately half of their electrons, some approximation must be made to account for the additional electronic screening. The simplest approximation is to assume that the fission fragment has some constant charge number, which we shall call Z_{eff}, that is effective in the scattering process. Z_{eff} then replaces Z_1 in the definitions of α and χ_c. This model might be expected to describe the scattering process generally, but its limitation lies in the fact that the effective charge would not be a constant, but a function of the impact parameter in any single scattering event. There is, of course, a restriction on Z_{eff}; its value must lie between the ionic charge number and the mean nuclear charge number of the fission fragments, i. e., between 24 and 52.5.

The experimental results, with statistical errors indicated, and the scattering distributions calculated in this approximation according to Molière and NSW are shown in Fig. 2. The theoretical distributions have been normalized to the peak height of the experimental distributions, and Z_{eff} has been chosen, within the limits indicated above, to give the best possible agreement at $\Theta_{1/e}$, i. e., the value of Θ at which the distribution has decreased to $1/e$ times the peak height. A correction which increases the value of $\Theta_{1/e}$ by approximately 2% has been applied to the theoretical distributions to account for the imperfect collimation of the incident beam. The notation "Molière (Keil)" indicates that the distribution has been calculated with Molière's χ_α, but according to the plural scattering results of Keil et al. In view of the above mentioned uncertainty in applying these results, a Molière distribution, which should not be too much in error, is also shown for the case of Au (0.63 mg/cm^2) where $\Omega_0 = 7.63$.

The NSW distributions have been calculated with $\mu = 1.8$, the value that NSW found to give best agreement with the experimental results of Hanson et al. [16]. A calculation by NSW yielded $\mu = 1.12$, and using this value, Nigam and Mathur [17] found agreement between their calculated difference in positron and electron multiple scattering and the difference determined experimentally by Henderson and Scott [18]. The present calculations are not very sensitive to the choice of μ, but the larger value yields distribution widths slightly closer to our experimental results.

To give an indication of the sensitivity of the $\Theta_{1/e}$ dependence on Z_{eff}, this dependence is shown graphically in Fig. 3. In cases where $\Omega_0 < 20$, $\Theta_{1/e}$ has been calculated only according to Keil et al. The horizontal line labelled "$\Theta_{1/e}$ (exp.)" extends from the value of the ionic charge of the fission fragment to the value of the mean nuclear charge. Agreement between theory and experiment exists, therefore, only if the $\Theta_{1/e}$ vs. Z_{eff} curve intersects this line.

It is seen from these figures and Table 1, that in all cases Molière's theory is able to predict distributions in agreement with experiment for Z_{eff} in the range from 41 to 49. The NSW theory, on the other hand, would require Z_{eff} values of 13 and 16 to yield the experimental widths for the two gold foils where α is the largest. As α decreases through the decreasing value of the atomic number of the scattering material, marginal agreement can be obtained for Ag, and the value $Z_{\mathrm{eff}} = 27.0$ gives agreement in the case of Formvar. Better agreement with decreasing α is of course expected since the two theories converge for small α.

[14] E. Konecny, H. Opower, H. Gunther, and H. Göbel, IAEA Report on Physics and Chemistry of Fission, Vol. I, 401 [1965].

[15] H. Opower, E. Konecny, and G. Siegert, Z. Naturforschg. **20 a**, 131 [1965].

[16] A. O. Hanson, L. H. Lanzl, E. M. Lyman, and M. B. Scott, Phys. Rev. **84**, 634 [1951].

[17] B. P. Nigam and V. S. Mathur, Phys. Rev. **121**, 1577 [1961].

[18] C. Henderson and A. Scott, Proc. Phys. Soc. London **A 70**, 188 [1957].

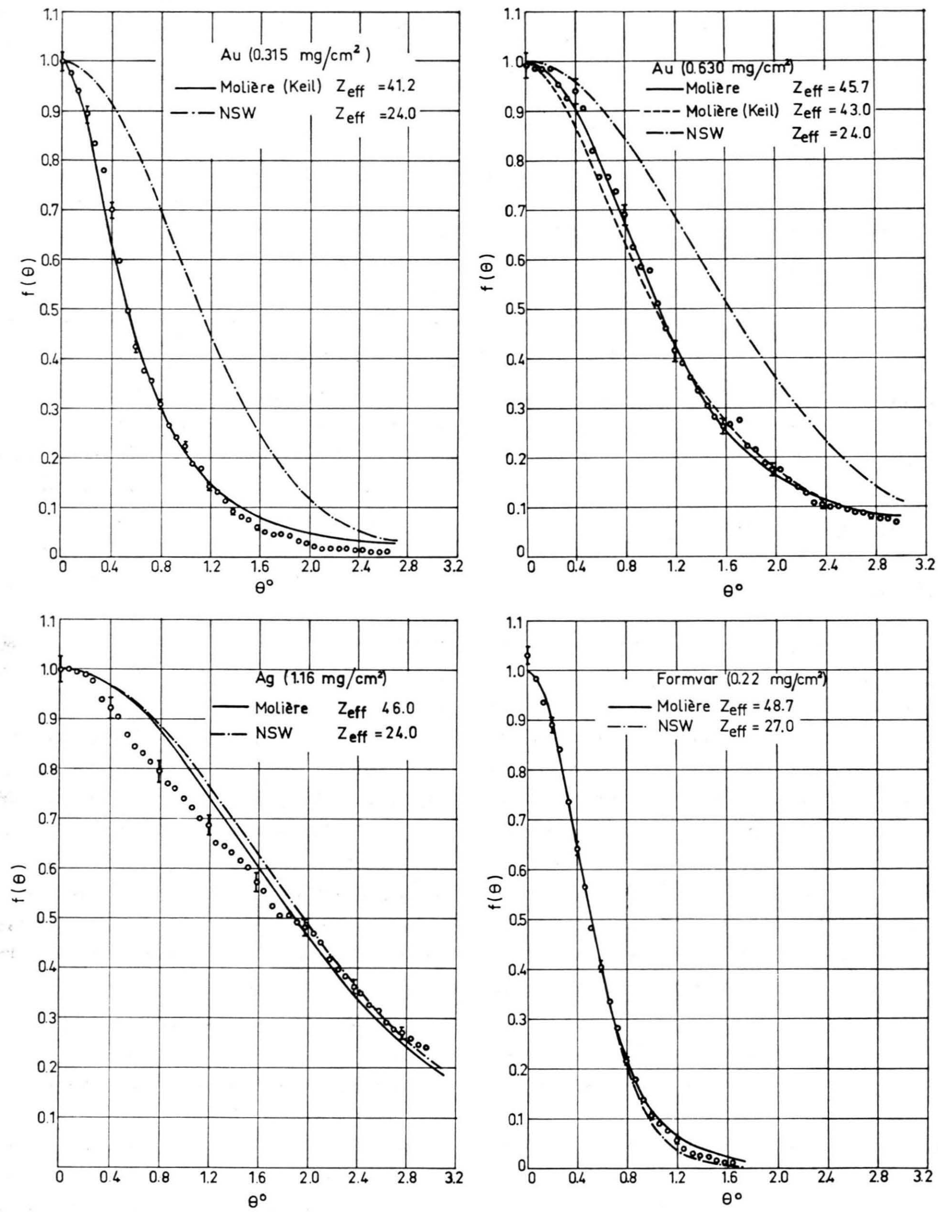

Fig. 2. The experimental multiple scattering distributions and the distributions calculated according to the theories of MOLIÈRE and NSW with the assumption that the fission fragments have an effective charge number, Z_{eff}, to account for their incomplete ionization.

2. The electronic screening of the fission fragment nucleus

A better approximation to account for the incomplete ionization of the fission fragments, and one having more physical significance, is to modify the screening in the potentials given in Eq. (1) and (6). In these potentials, which are applicable only if the impinging particle is completely ionized, the THOMAS–FERMI radius may be written as

$$r_0 = 0.885\, a_0/s \tag{13}$$

where

$$s = Z_2^{1/3}. \tag{14}$$

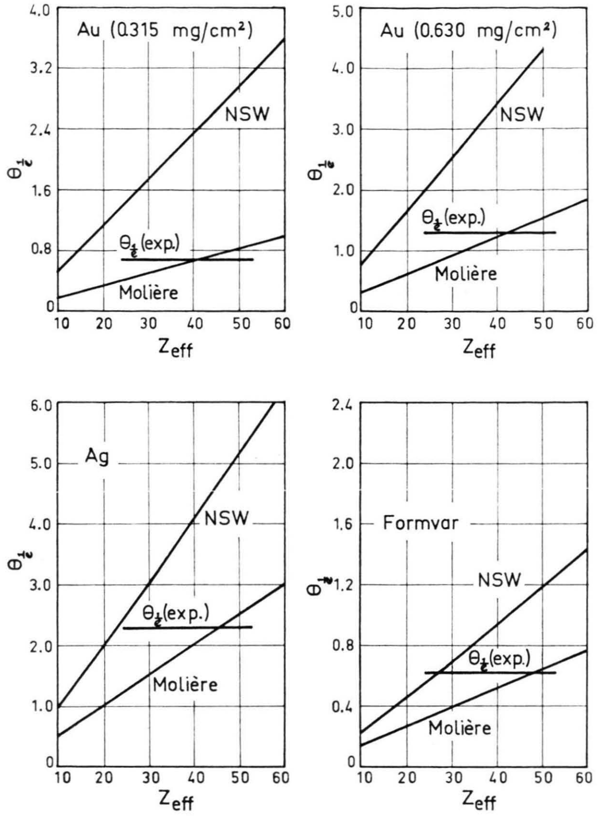

Fig. 3. The width of the multiple scattering distributions calculated according to the theories of MOLIÈRE and NSW as a function of Z_{eff}. $\Theta_{1/e}$ is the value of Θ at which the distribution has decreased to $1/e$ times the peak height. The line labelled "$\Theta_{1/e}$(exp.)" represents the experimentally determined width and is drawn between the limits of Z_{eff} given in the text.

According to BOHR [19], if colliding atoms with nuclear charge number Z_1 and Z_2 have individual screening given by Eq. (14), then the combined screening can be represented approximately by

$$s = (Z_1^{2/3} + Z_2^{2/3})^{1/2}. \tag{15}$$

Since the fission fragments are partially ionized, the screening should be given by some value of s lying between $Z_2^{1/3}$ and $(Z_1^{2/3}+Z_2^{2/3})^{1/2}$. This correction can be introduced by multiplying the existing term in the exponent of either of the potentials by some factor F with a value between 1 (fission fragment fully ionized) and $(Z_1^{2/3}+Z_2^{2/3})^{1/2}/Z_2^{1/3}$ (fission fragment not ionized).

The potential used by MOLIÈRE then becomes

$$V_F(r) = \frac{Z_1 Z_2}{r}\, e^2 \sum_{i=1}^{3} a_i \exp(-F\, b_i\, r/r_0) \tag{16}$$

[19] N. BOHR, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **18**, 8 [1948].

and χ_0 is replaced by

$$\chi_{0F} = F\,\hbar/p\,r_0\,. \tag{17}$$

When the factor F is included in MOLIÈRE's calculation of χ_α, the expression giving this parameter becomes

$$\chi_{\alpha F}^2 = \chi_{0F}^2\, F^2 (1.13 + 3.76\,\alpha^2) = F^4\, \chi_0^2 (1.13 + 3.76\,\alpha^2). \tag{18}$$

In the case of the exponentially screened potential used by NSW, we need only replace μ by $F\,\mu$ wherever it appears and to adjust χ_0 as in Eq. (17). The NSW expression for the screening angle is then

$$\chi_{\alpha F}^2 = F^4\,\chi_\mu^2\{1 - 4\,\alpha\,F^2\,\chi_\mu[(1-\beta^2)\ln(F^2\,\chi_\mu) + 0.2310 + 1.448\,\beta^2]\}\,. \tag{19}$$

Since the maximum value of F is different for each scattering material, we write it in the form

$$F = 1 + f\left[\frac{(Z_1^{2/3}+Z_2^{2/3})^{1/2}}{Z_2^{1/3}} - 1\right] \tag{20}$$

where $0 \leqq f \leqq 1$. The factor f, which is the ratio of the actual increase in screening to the maximum allowed increase, is used for comparison purposes.

Scattering distributions calculated on this basis and fitted within the limits of f to give the best agreement with experiment are shown in Fig. 4. Where appropriate, the results of KEIL et al. have been used, and, as before, we have taken $\mu = 1.8$ for the NSW curves. KEIL et al. have calculated an exact distribution for $\Omega_0 = 20$ and therefore, in the case of Ag ($\Omega_0 = 20.0$), this result has been applied.

Fig. 5 shows the experimentally determined values of $\Theta_{1/e}$ and the dependence of $\Theta_{1/e}$ on f according to the two theories for all the scattering foils. The theoretical and experimental results for Au (0.315 mg/cm^2) require special comment. The dependence of $\Theta_{1/e}$ on f, calculated with MOLIÈRE's χ_α but with the distributions of KEIL et al., appears unusual in that $\Theta_{1/e}$ increases after reaching a minimum at $f \cong 0.4$. This may be understood if one considers the two opposing effects which influence the width of a scattering distribution. As the screening increases, the scattering angle in a single scattering event becomes generally larger tending to broaden the distribution. At the same time, Ω_0, which is taken to be the average number of collisions, decreases, tending to make the distribution narrower. When Ω_0 is reasonably large, the latter effect dominates. However, when Ω_0 is of order unity, it has little influence on the distribution of *scattered* par-

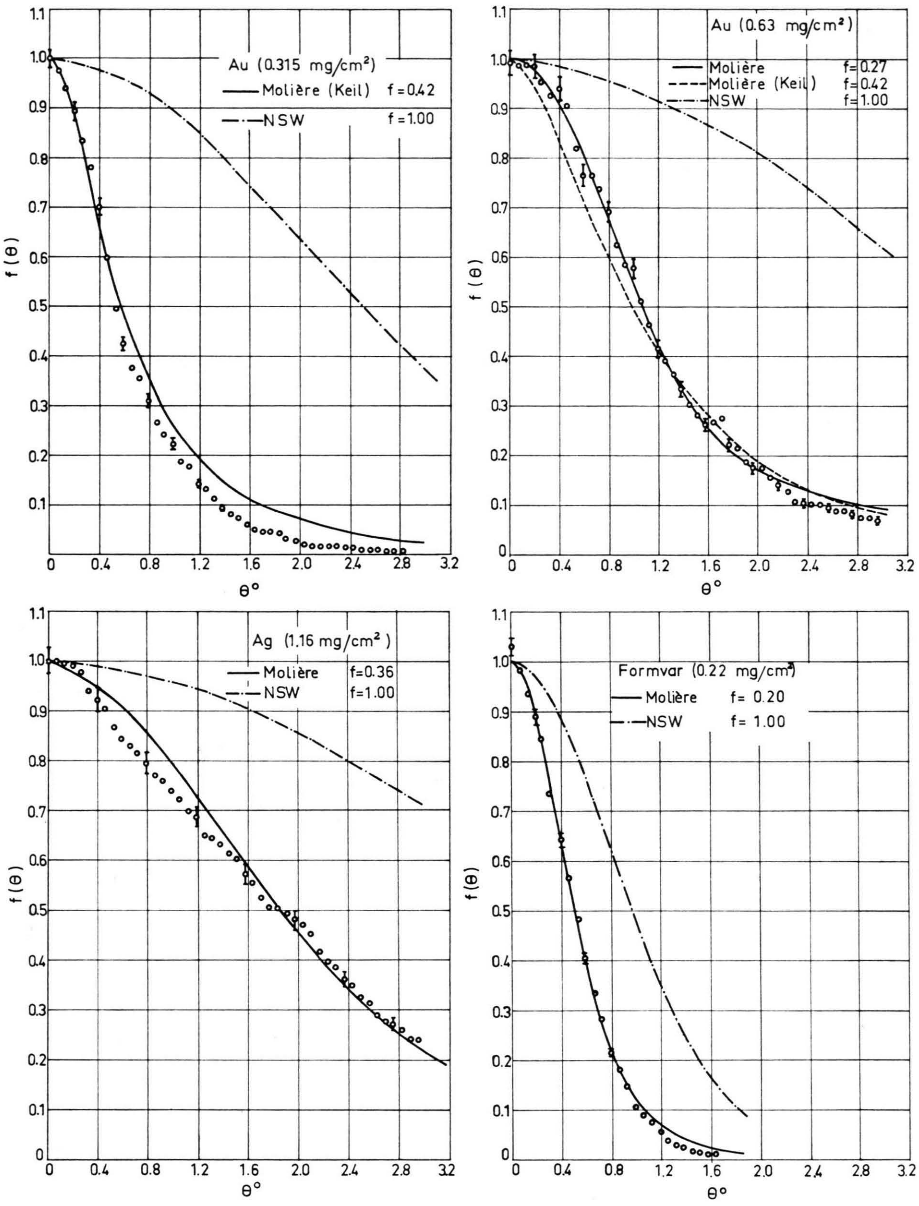

Fig. 4. The experimental multiple scattering distributions and the distributions calculated according to the theories of MOLIÈRE and NSW with the potential employed in each theory adjusted to include the effect of the electronic screening of the fission fragments.

ticles, since most particles are either undeflected or are scattered only once; only the percentage of scattered particles decreases with decreasing Ω_0. Therefore, at some point the first effect must become dominant causing the distribution width to increase.

Regarding the experimental result for Au (0.315 mg/cm^2), it was pointed out in a previous section that the value of Ω_0 is probably less than the true number of scattering events. This is particularly evident in this case, where $\Omega_0 = 2.31$ when the theoretical value of $\Theta_{1/e}$ is closest to the experimental result. This value of Ω_0 implies that 10% of the beam pases through the foil unscattered, but the shape of the experimental distribution places an up-

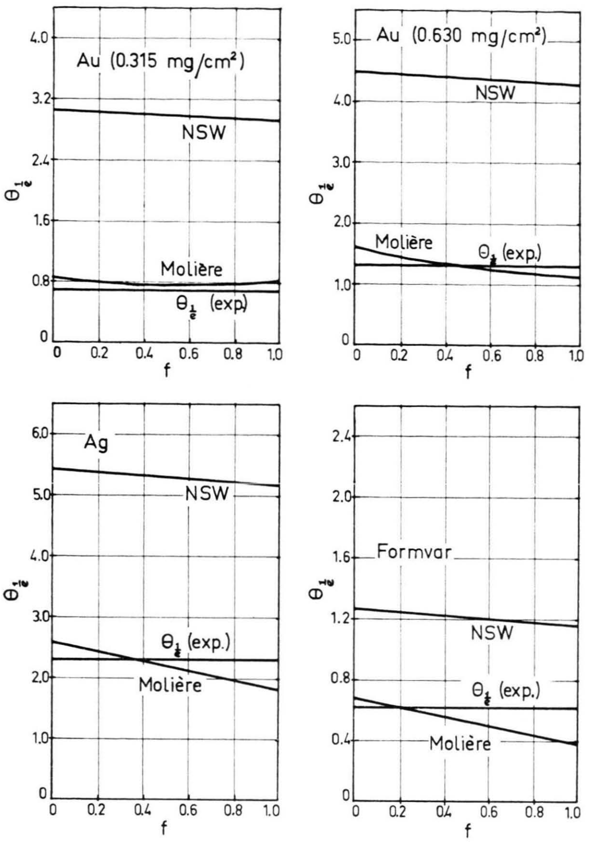

Fig. 5. The width of the multiple scattering distributions calculated according to the theories of MOLIÈRE and NSW as a function of f (see text for definition). $\Theta_{1/e}$ is the value of Θ at which the distribution has decreased to $1/e$ times the peak height. The line labelled "$\Theta_{1/e}$(exp.)" represents the experimentally determined width.

per limit of 2% on the contribution of the unscattered beam, or a lower limit of 4 on the average number of scattering events. If the experimental distribution is corrected to account for this percentage of unscattered particles, then agreement can be obtained between MOLIÈRE's theory and experiment.

From Figs. 4 and 5, and from Table 1, which summarizes the experimental and theoretical results, it is clear that the NSW theory fails for the large values of α involved in these measurements. The distributions predicted using the most favourable, but unrealistic, value of f (its upper limit) are from 2 to 4 times too wide. The MOLIÈRE theory, however, predicts distribution widths in agreement with those determined experimentally for values of f that are not unreasonable for particles having approximately half their electrons. (The somewhat low value of f for Formvar is questionable since the foil thickness in this case could only be measured with an accuracy of 20%.)

Although the results according to MOLIÈRE's theory agree with experiment for both the approximations we have considered, the adjustment of the screening term in the potential is the better and more realistic method. It has the advantage that, for a given set of parameters, the limits on f restrict the value of the distribution width to a considerably smaller interval than do the limits on Z_{eff} in the approximation assuming an effective charge for scattering. Also, the value of f should be more closely related to the degree of ionization than should Z_{eff}.

In the present case, the fission fragments have retained approximately 50% of their electrons, and the mean value of f (neglecting the uncertain value for Formvar) is 0.4, indicating a reasonable correlation between this parameter and the degree of ionization. Therefore, MOLIÈRE's theory with the modified potential, together with the correlation indicated here, may be of some use in estimating scattering distributions in other experiments involving partially ionized particles.

An examination of the results of multiple scattering experiments (see Ref. [7] for a summary of previous works) shows that MOLIÈRE's theory gives generally good agreement for all the involved values of α [20]. For small α (electron scattering), the agreement is not exact, but, at the same time, the NSW theory predicts agreement through the adjustment of the parameter μ. In this region of small α it is therefore difficult to say which of the theories is the more accurate. The results of BEDNYAKOV et al. and of SIMON verify the validity of MOLIÈRE's theory [21] for values of α up to 30, and SIMON shows that the NSW theory has already begun to fail for $\alpha \sim 3$.

[20] In a recent publication, J. B. MARION and B. A. ZIMMERMAN, Nucl. Instr. Methods **51**, 93 [1967], compare multiple scattering distributions calculated according to the theories of MOLIÈRE and NSW and seem to recommend the NSW theory. They place no limit on the range of α for which it may be used, and suggest, in particular, that it may be applied to the scattering of heavy ions, in contradiction to the present results and those of SIMON.

[21] Note added in proof: M. ROGGE at the Max Planck Institute for Physics, Heidelberg, has also found agreement between MOLIÈRE's theory and an experiment in which heavy ions from an accelerator were scattered (unpublished, private communication).

Scattering foil	Au	Au	Ag	Formvar
Foil thickness				
(mg/cm^2)	0.315	0.630	1.16	0.22
$\Theta_{1/e}$ (experim.)	0.69 ± 0.03	1.31 ± 0.05	2.30 ± 0.05	0.62 ± 0.02
Molière (with Z_{eff})				
Z_{eff}	41.2	43.0	46.0	48.7
α	666	695	442	(169)*
χ_α	0.299	0.318	0.185	0.041
Ω_0	3.86	7.63	36.1	49.0
$\Theta_{1/e}$	0.69	1.31	2.30	0.62
NSW (with Z_{eff})				
Z_{eff}	24.0	24.0	24.0	27.0
α	388	388	231	(94)*
χ_α	4.41×10^{-4}	4.51×10^{-4}	4.02×10^{-4}	2.50×10^{-4}
Ω_0	5.97×10^5	1.18×10^6	2.09×10^6	3.94×10^5
$\Theta_{1/e}$	1.34	1.98	2.38	0.62
Molière (with additional screening)				
f	0.42	0.42	0.36	0.20
α	848	848	505	(182)*
χ_α	0.491	0.502	0.284	0.060
Ω_0	2.31	4.56	20.0	26.3
$\Theta_{1/e}$	0.77	1.31	2.30	0.62
NSW (with additional screening)				
f	1.00	1.00	1.00	1.00
α	848	848	505	(182)*
χ_α	8.88×10^{-4}	9.07×10^{-4}	9.14×10^{-4}	8.73×10^{-4}
Ω_0	7.05×10^5	1.39×10^6	1.94×10^6	1.22×10^5
$\Theta_{1/e}$	2.93	4.30	5.17	1.16

* α cannot be defined for a mixture of elements. The largest individual α involved is quoted for comparison purposes.

Table 1. Summary of the experimental results and the results calculated with the theories of Molière and NSW according to the two models described in the text. Z_{eff} and f have been chosen within their limits to give the best agreement between the theoretical and experimental values of $\Theta_{1/e}$. Note that when the approximation involving Z_{eff} is used, α is actually an "effective" α directly dependent on Z_{eff}. All angles are in degrees.

The present work shows that Molière's theory still predicts reasonable results for α as large as 850, while the predictions of the NSW theory and the experimental results continue to diverge for very large α.

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